that the modal amplitudes may be readily obtained without having to invert $\langle L^*, L\rangle$ for each α.

The matrices in (5) can be written with the α dependence made explicit in the following way:

$$(M_1 + \alpha M_2)V = (f_1 + \alpha f_2). \qquad (6)$$

Since M_1 and M_2 are Hermitian, the weighted eigenvalue equation

$$M_2 v_i = \lambda_i M_1 v_i$$

has real eigenvalues, and its eigenfunctions are orthogonal with respect to the weights M_1 and M_2. Then, substituting

$$V = \sum \beta_i v_i$$

into (6) and multiplying the resulting equation by v_j^* to obtain β_j, we get

$$V = \sum_i \frac{v_i^*(f_1 + \alpha f_2)}{1 + \alpha\lambda_i} v_i$$

assuming normalized eigenvectors (divide v_i by $(v_i^* M_1 v_i)^{1/2}$). A similar expression can be obtained by expanding f_1 and f_2 [10]. Once λ_i and v_i are computed we may obtain V for different values of α. We may also obtain an estimate for α by substituting the propagating modal amplitudes into the expression of the conservation of real power and imposing approximations $|\alpha\lambda_i| \ll 1$ and/or $|\alpha\lambda_i| \gg 1$. Other expressions for V are given in [10].

Conclusion

A numerical method for the solution of waveguide discontinuities has been developed here which is suitable for computer implementation and which does not suffer from some of the shortcomings of the other methods. We have solved many problems numerically which do not appear in the extant literature. The method can as well handle other types of waveguides and discontinuities.

The problem that remains to be solved is that of an easier criterion for the selection of the weighting factor (α) so that the smallest possible number of modes can be used for a given accuracy. Davies [5] uses one among several condition numbers of the matrices as the criterion for the selection of α. Such a condition number may be an indicator of the stability of the matrix inversion, but its relation to the equivalent susceptance of the discontinuities and the dominant modal amplitudes remains obscure.

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The Effect of Surface Metal Adhesive on Slot-Line Wavelength

JEFFREY B. KNORR AND JUAN SAENZ

***Abstract*—An investigation of the dependence of slot-line wavelength upon a thin layer of adhesive between metal and substrate is described. It is shown that the presence of adhesive will cause an increase in wavelength when the dielectric constant of the adhesive is less than that of the substrate. Experimental results are presented which show this dependence for a variety of surfaces and adhesives. A perturbation expression is given which permits correction of experimental data for comparison with theory when this effect occurs.**

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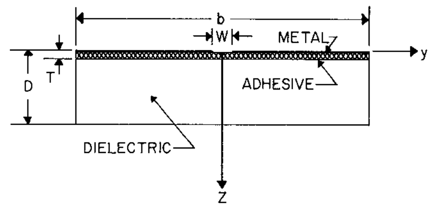

Fig. 1. Slot-line geometry with adhesive.

TABLE I

THICKNESS OF METALS AND ADHESIVES

Metallization	Thickness of Metal (Mils)	Thickness of Adhesive (Mils)
Factory 1 Oz. Copper	1.15	< 1.0
3M Copper Tape	1.25	1.9
3M Aluminum Tape	1.95	1.8
Circuit-Stik Copper Foil	1.15	2.3
Evaporated Copper	0.65	0

I. Introduction

The analysis of slot line and its microwave applications have been discussed by a number of authors [1]–[7] during the past several years. In one of these papers, Mariani *et al.* [5] presented measured values of slot wavelength for various substrates metallized with both aluminum sensing tape and copper (electroless plated). Their data showed that the slot wavelength on substrates metallized with aluminum sensing tape exceeded the theoretical value. For substrates with copper plated surfaces, the measured wavelength was (with one exception) somewhat less than the theoretical wavelength. It was concluded that the adhesive which was present in the case of aluminum sensing tape decreased the effective dielectric constant and thereby increased slot wavelength.

Measurements in our laboratory substantiate this conclusion. The purpose of this short paper is to present more consistent and extensive data on this effect and to treat the problem using perturbation theory.

II. Slot-Wavelength Measurements

Slot line is constructed by etching a slot utilizing a dielectric substrate which has been metallized on one side only. The metal may be applied in various ways and in some cases a thin layer of adhesive is present between the metal and the substrate as illustrated in Fig. 1. This adhesive may have a significant effect upon the slot wavelength.

A number of experiments were conducted to investigate adhesive effects. In one series of experiments a Custom Materials Hi-K707-20 ($\epsilon_r = 20$) substrate was tested using several different methods of metallization. The substrate was 3-in wide by 0.125-in thick, and slot width was maintained constant in all cases with $W/D = 0.53 \pm 0.02$. The surfaces tested were 1-oz copper as supplied by the manufacturer, 3M copper tape (1-in wide), 3M aluminum tape (1-in wide), and a vacuum deposited copper surface. Table I lists the thicknesses of metal and adhesive for all surfaces tested.

Measured values of λ'/λ for these surfaces are displayed in Fig. 2 along with the theoretical curve from [5]. The vacuum deposited copper surface is in intimate contact with the substrate and the wavelength ratio for this surface may be used as a basis for comparison of experimental measurements. All other surfaces are separated from the substrate by an adhesive layer and increased wavelength ratios result.

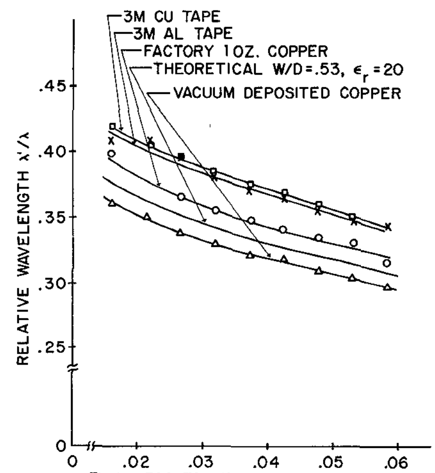

Fig. 2. Theoretical and experimental values of λ'/λ versus D/λ for various surfaces and adhesives.

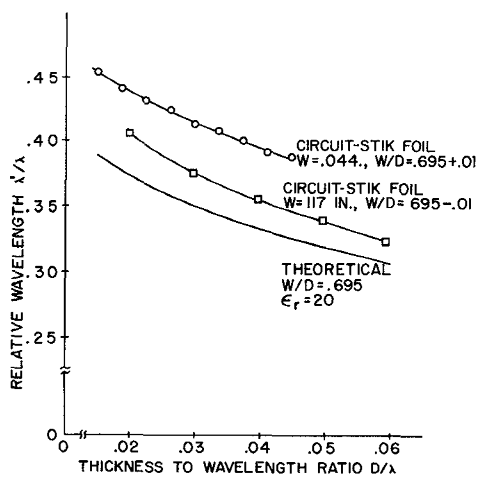

Fig. 3. Theoretical and experimental values of λ'/λ versus D/λ for copper foil.

The discrepancy between theory and experiment for the vacuum deposited copper surface is about 3.5 percent. This could be partially caused by a difference between the actual value of dielectric constant and the assumed value of 20. The tolerance of ± 4.5 percent on ϵ_r results in an uncertainty of about ± 2.25 per cent in λ'/λ. The agreement obtained, however, is consistent with other reported measurements [5]. Theoretical results obtained by the two very different methods of Cohn [1] and Itoh and Mittra [4] agree to within a few tenths of a percent which seems to support the accuracy of the theory.

In a second series of experiments, Hi-K707-20 substrates were covered with copper foil supplied by Circuit-Stik. This foil had a nonconductive adhesive backing. Two substrates were tested with $W/D=0.695\pm 0.01$. The first was 0.117-in thick by 3.6-in wide. The slot wavelength was found to exceed the theoretical value by 6–10 percent over the 2–6-GHz frequency range. The second substrate was machined to a thickness of 0.044 in and was 1.9-in wide. Measurements from 4–12 GHz revealed wavelengths 16–18 percent in excess of those theoretically predicted. Fig. 3 shows both theoretical [5] and experimental curves of wavelength for these substrates.

Adhesive thickness remained constant during the preceding measurements but the ratio of adhesive to substrate thickness did not. Fig. 3 shows a much greater discrepancy between theory and experiment in the case of the thinner substrate. A plot of $\Delta\lambda/\lambda_{\text{theoretical}}$ versus the adhesive to substrate thickness ratio T/D shows that the effect is linearly dependent on T/D. This is illustrated in Fig. 4. Perturbation theory predicts such a dependence as will be shown shortly.

In all of the experiments described here, measurements were recorded using a custom-built test fixture. This fixture supported the slot line and a probe constructed from 0.085-in semirigid coaxial.

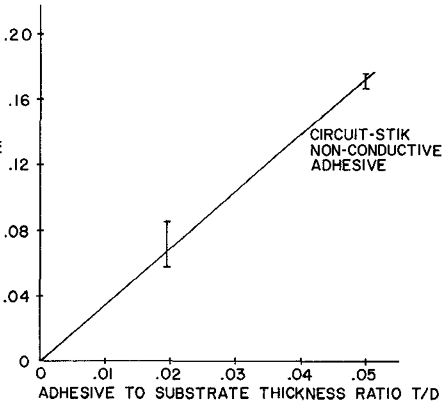

Fig. 4. Wavelength dependence on adhesive thickness.

The probe was placed on the air side of the metal and was used to sample the electric field component parallel to the metal surface. Distance along the slot was measured to the nearest tenth of a millimeter. All slots were photoetched and all line dimensions were recorded with an accuracy greater than 10^{-4} in.

The manufacturing tolerance of the Custom Hi-K707-20 is $\epsilon_r=20\pm 0.9$. For the measurements of Figs. 3 and 4, substrate dielectric constant was checked by the cavity perturbation technique and found to be within 2.5 percent of the specified value. For other measurements the dielectric was assumed to be within the tolerance specified by the manufacturer.

III. Perturbation Analysis

The adhesive effect may be treated using perturbation theory. An appropriate expression is

$$\beta' - \beta = \omega \frac{\int\int_{\Delta S} \Delta\epsilon E' \cdot E^* \, da}{\int\int_S (E^* \times H' + E' \times H^*) \, da} \tag{1}$$

which may be found as equation (7–30) of [8].[1] In this expression, the primed and unprimed quantities refer to perturbed and unperturbed values, respectively, while $\Delta\epsilon$ is the change in permittivity.

For the problem at hand, we rewrite (1) as

$$\frac{\Delta\lambda}{\lambda_{th}} = -\int_{-b/2}^{+b/2}\int_0^T \frac{\Delta\epsilon E' \cdot E^* \, dydz}{4P_{av}}. \tag{2}$$

E' would be found from E in the usual way, taking into account depolarization. Now, the integrand in (2) may be assumed constant over the small distance T as we integrate with respect to z and thus

$$\frac{\Delta\lambda}{\lambda_{th}} = -\frac{2T\Delta\epsilon \int_0^{b/2} E'(y,0)\cdot E^*(y,0)\, dy}{4P_{av}}. \tag{3}$$

For any given frequency and line constants, the integration in (3) may be carried through to give some constant times the square of the slot voltage, V_0. Thus

$$\frac{\Delta\lambda}{\lambda_{th}} = -\text{CONST} \times T\Delta\epsilon Z_0 \tag{4}$$

where we have used Cohn's definition of impedance, $Z_0 = V_0^2/2P_{av}$ [1].

Now applying the scaling principle, if the wavelength is changed from λ_1 to λ_2 and all dimensions are scaled by λ_2/λ_1, then the electrical properties of the circuit remain unchanged. On the other hand, if all dimensions and wavelength are scaled and then adhesive thickness is allowed to increase, the change in $\Delta\lambda/\lambda_{th}$ will be in direct proportion to the change in the relative adhesive thickness T/D according to (4). This is clearly shown in Fig. 4.

Several other observations should be made with regard to Fig. 4. First, the spread of data corresponds to the variation observed over a frequency range of 2–6 GHz in one case and 4–12 GHz in the other. We

[1] While Harrington gives (1) for a cylindrical waveguide, it is easy to show that it also holds for structures on which the fields decay with transverse coordinates.

conclude that any frequency dependence is very small. Second, the data show $\Delta\epsilon$ negative so the dielectric constant of the adhesive was less than that of the substrate. It was known in this case that the adhesive had a dielectric constant of 3.25 at low frequencies.

IV. Conclusions

It has been shown that the wavelength in slot line is sensitive to the dielectric constant and thickness of any adhesive present between the substrate and the conducting surface. If the dielectric constant of the adhesive is less than that of the substrate, wavelength increases and this increase is in direct proportion to the ratio T/D.

While adhesive effect would normally be considered undesirable, it is possible by use of the simple expressions developed here to correct experimental data for comparison with theory without having detailed knowledge of the properties of the adhesive (Fig. 4).

Acknowledgment

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The Limiting Value of the Interaction Between Symmetrical Fringing Capacitances

HENRY J. RIBLET

***Abstract*—It is well known that the fringing capacitances determined for rectangular bars between parallel plates interact with each other when $w/(b-t)\to 0$. The limit of this interaction as $s\to 0$ for fixed w, b, and t is determined for symmetrical odd-mode fringing capacitances. This limit, together with an exact value known from one rectangular section and the known asymptotic value as $s\to 0$, permits one to estimate the values for all s. The same is true for the interaction of the symmetrical even-mode fringing capacitances, except that their interaction is readily shown to tend to zero as $s\to 0$.**

If we denote by C_0 the total capacitance of a structure of unit length whose cross section is shown in Fig. 1, then the exact odd-mode fringing capacitance C_{f_0}, is defined by the equation $C_0=4C_{f_0}+2C_p$ where C is the parallel plate capacitance associated with the side of the inner conductor whose length is W_0. We have then $C_p=2W_0/(B_0-T_0)$.

On the other hand, the "approximate" odd-mode fringing capacitance, C_{f_0}',[1] is defined as half the limit of the difference between the total capacitance and the parallel plate capacitance of the structure, shown in Fig. 2, as the magnetic wall recedes to infinity at the right.

If we denote $C_{f_0}'-C_{f_0}$ by ΔC_{f_0}, then this short paper is concerned in a general way with the evaluation of ΔC_{f_0} for given W_0, B_0, and T_0 as a function of S; and, in particular, with the value of $\bar{\Delta} C_{f_0}$, the limit of ΔC_{f_0} as $S\to 0$. The special interest in the value of $\bar{\Delta} C_{f_0}$ arises from the fact that exact values of ΔC_{f_0} are already known for $S=\infty$ and at one intermediate point [2]. From an accurate estimate for ΔC_{f_0}, one may then determine the value of C_{f_0}. This quantity undoubtedly is more accurate for general design purposes than C_{f_0}', since it is exact in the symmetrical case regardless of any interaction.

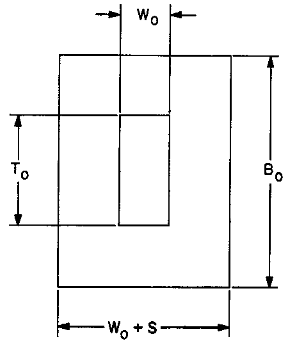

Fig. 1. Geometry defining C_{f_0}.

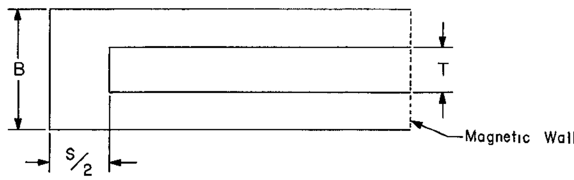

Fig. 2. Geometry defining C_{f_0}'.

It is not difficult, following Bowman [3], to express the quantities B, S, and T of Fig. 2, except for a scale factor, in terms of two independent real parameters, a and k, where k is the modulus of the Jacobi elliptic functions involved. It is no restriction to assume that $0<k<1$ and that $0<a<K$. Then we have

$$B = 2K'\left\{\frac{\operatorname{sn} a \operatorname{dn} a}{\operatorname{cn} a} - Z(a)\right\} - \frac{\pi a}{K} + \pi \tag{1}$$

$$S = 2K\left\{\frac{\operatorname{sn} a \operatorname{dn} a}{\operatorname{cn} a} - Z(a)\right\} \tag{2}$$

$$T = 2K'\left\{\frac{\operatorname{sn} a \operatorname{dn} a}{\operatorname{cn} a} - Z(a)\right\} - \frac{\pi a}{K}. \tag{3}$$

The approximate odd-mode fringing capacity, C_{f_0}', for this geometry is given in terms of the same parameters, a and k, by the expression[2]

$$\pi C_{f0}' = 2(K-a)\left\{\frac{\operatorname{sn} a \operatorname{dn} a}{\operatorname{cn} a} - Z(a)\right\} - 2\log(k \operatorname{sn} a \operatorname{cn} a) - 4\log(\theta_n(a)). \tag{4}$$

Here the functions are all those which are familiar from Jacobi's theory of elliptic functions, but it may be well to recall that $\theta_n(a)=\Theta(a)/\Theta(0)$.

It is clear from (2) that $S\to 0$ as $a\to 0$. If now T/B is to approach the finite limit, T_0/B_0, as $a\to 0$, then T must approach a finite limit >0 as $a\to 0$. This can only happen if $K'\to\infty$ and, in turn, $k\to 0$. We determine then the limit of Cf_0' as a and $k\to 0$; and, for this, we will need the precise relationship of a and k in this limit.

To this end, we write down the expansions of the various elliptic quantities occurring in (1), (2), and (3) in ascending powers of a and k. Thus

$$K = \frac{\pi}{2}\left(1+\frac{k^2}{4}+\cdots\right)$$

$$K' = \left(1+\frac{k^2}{4}+\cdots\right)\log\frac{4}{k} - \frac{k^2}{4}+\cdots$$

$$\operatorname{sn}(a,k) = a - \frac{1+k^2}{6}a^3+\cdots$$

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[1] This is equivalent to the Cf_0' used by Getsinger [1], and it should also be noted that the geometrical capacitances of this short paper must be multiplied by the permittivity to obtain the true capacitances.

[2] This formula is somewhat simpler than the one given by Getsinger [1], to which it may be presumed to be equivalent on the basis of a comparison of numerical results.